

Let  $\hat{m}$  be the maximum likelihood estimator of the offspring mean  $m$  given by

$$\hat{m} = \frac{\sum_{n=0}^N X_{n+1}}{\sum_{n=0}^N X_n}.$$

Using some standard results due to Jagers (1975) and Dion (1972) on branching processes, the following limit theorem is established in this paper.

**Theorem 1.** *Under  $P(\cdot | E_0^c)$ , where  $E_0^c$  denotes the complement of  $E_0$ , the set of nonextinction,*

$$\left( \left( \sum_{n=0}^N X_n \right)^{-1/2} \left( \sum_{n=0}^N X_{n+r} - \hat{m}^r \sum_{n=0}^N X_n \right), r = 2, 3, \dots, T \right)$$

*converges in law, as  $N \rightarrow \infty$ , to a normal vector with mean zero and a well specified covariance matrix.*

This is in line with the similar results due to Venkataraman and Nanthi (1982) based on the Lotka–Nagaev estimator.

A parameter free limit theorem based on Theorem 1 leading to a limiting  $\chi^2$ -statistic is also derived in this paper which will find its use in the area of the asymptotic goodness of fit tests for a supercritical Galton–Watson process.

## On a Modified Markov Branching Process II

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A Markov branching process is modified in such a way that only particles of age lying in the age interval  $(T_1, T_2)$  can give rise to offspring. An exact expression for the mean number of particles living at time  $t$  is given. The extinction probability of the process is computed and is shown to be greater than that for a Bellman–Harris process without any restriction.

## Maximum Likelihood Estimation for a Multitype Continuous-Time Branching Process with Immigration

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We consider a multitype Markov branching process with immigration  $\{X_t; t \geq 0\}$ . Maximum likelihood estimators for several parameters of this process are obtained and their asymptotic properties as  $T \rightarrow \infty$  are studied when the process  $\{X_t; t \geq 0\}$  is observed continuously over a fixed interval  $[0, T]$ .